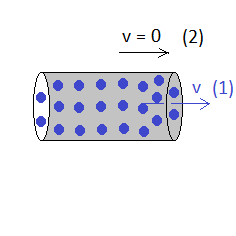
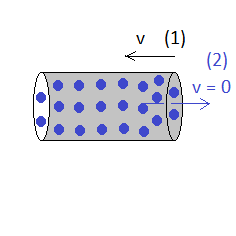
**Viscosity**

So we know that below Tc, He is a liquid consisting of two parts: a normal fluid and a superfluid. The normal fluid is viscous, whereas the superfluid/condensate is not. Landau made an argument that kind of explains the presence of a critical velocity below which the condensate part of the fluid would superflow. So consider a superfluid flowing with velocity v down a pipe at rest with velocity 0. The superfluid would slow down, i.e., ‘experience’ viscosity, if it transferred some of its energy/momentum to the pipe. Specifically, I guess it would transfer some of its bulk kinetic energy into its own internal energy and some of its bulk kinetic energy into the pipe’s internal energy + bulk kinetic energy. Some of the fluid’s bulk kinetic energy would have to go into internal energy due to the 2nd law because entropy would have to increase, since it’s an irreversible process I think. Well this means that it must have an internal excitation structure to allow for such a transfer.



It’s easier to analyze the problem from a different point of view. Let’s jump to the reference frame of the fluid. Then it’s at rest, and the pipe is flowing backwards with velocity -v. Then in order for the pipe to ‘slow’ the fluid down (or, from this frame of reference, accelerate it leftward) it must induce an excitation in the presently stationary fluid. And like we said before, some of this energy must go into internal energy.



So the question is then, can we have a situation where we have an initially stationary fluid with a pipe coming along with initial velocity v (or -v whatever), ‘colliding’ with the fluid, and inducing an internal energy/momentum carrying excitation into the fluid, and obviously inducing a change in energy/momentum within itself (the pipe), by conservation of energy/momentum? It would seem like the answer is unequivocally yes, but it is in fact equivocal.

When, in the Classical Mechanics Folder, we analyzed collisions of a particle (1) against a stationary particle (2), we found in the limit m1 >> m2, that v1f ≈ v1i, and that 0 < v2f < 2v1i. Let’s revisit the same question, but this time, we’ll say that we have a particle (1) , i.e., the pipe, which induces some excitation in particle (2), i.e., the fluid, with energy ε2 and momentum p2. What are the restrictions on ε2 and p2? Well, our two conservation equations are:



now dot the momentum conservation equation with itself.



Subtracting the two tells us that:



In the limit of large m1 this comes to:



So we have, considering that v1f ≈ v1i.



So if the quantity ε2/p2 has a minimum value, vcritical,



then this would say that excitations can only be generated for v1i > vcritical, and so we’d have superflow for all tube speeds for which v1i < vcritical. So what if we have a free particle spectrum? Then we have:



as the minimum value v2 can have is 0, as v2 can be anything non-negative. So in this case, there is no speed at which we get critical flow. So I guess the normal fluid component has excitations that are mainly of free particle type? Tangentially, we’ll observe that our inequality becomes ε2/p2 < v1f 🡪 v2/2 < v1f → v2 < 2v1f≈ 2v1i, which coincides with what we found in the particle case in the Classical Mechanics Folder (generic collisions). What if we have a sound wave dispersion relation?



So we have a non-zero critical velocity in this case:

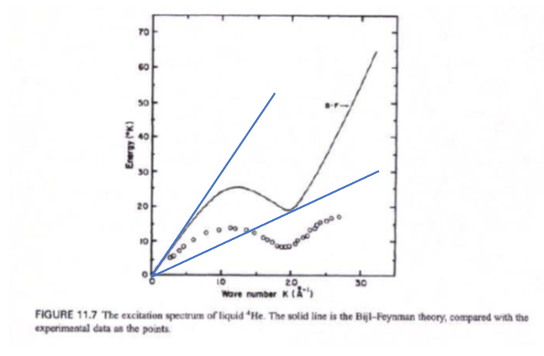


And so no excitation could be induced if the pipe were moving at a speed v1i < vs, the speed of sound within the fluid. Or from the other point of view, where it’s the pipe fluid moving and not the pipe, no excitation can be induced if the fluid moves with speed less than vs. If it moves with speed greater than this, then presumably the pipe would induce excitations within the fluid which would bump condensate particles into the normal fluid phase, and the fluid would slow down. There can be multiple minimums. Consider a more complicated spectrum ε2(p2).

Then ε2/p­2 is, geometrically, the slope of the line that connects the origin to a point on the curve. And this will have a minimum value, I guess for differentiable curves, where it not only touches the curve, but is tangent to it too, i.e., where: ε2/p2 = dε2/dp2. As can verify,



This is illustrated below – there is a minimum at the origin, and also near the roton minimum.



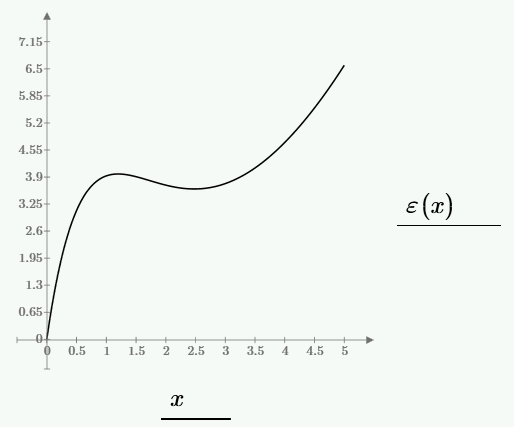
FWIW, it seems this estimate of the critical velocity is usually way too large. Apparently ‘some’ kind of excitations occur at lower velocities. But the basic idea is sound. Or maybe experiments are measuring the vcritical near the roton minimum.

**Example**

A Bose liquid has the excitation spectrum:



depicted below.



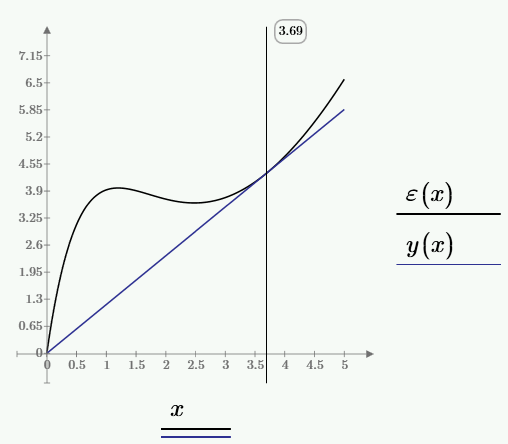
What is the maximum superfluid speed of the bose condensate near the roton branch of the spectrum? So we have to solve:



This comes out to:



Can graphically verify this. We’ll plot y(x) = ε(ln(40))x/ln(40) beside it.



So the critical speed is:



which gives us:



Another way is to just directly minimize ε/p,



So we form,



Then find the local minimum,



and then evaluate the velocity,



So that way might be easier. By comparison, the sound wave velocity at T = 0 would be the slope at p = 0:

